Chapter 23

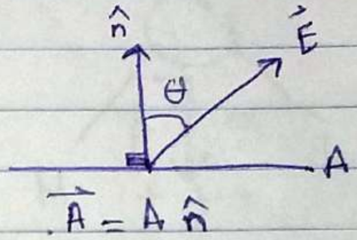
Gauss' law.

• **Electric Flux Φ_E**: The number of electric field lines crossing the surface perpendicularly for uniforme $\vec{E}$.

$$\Phi_E = \vec{E} \cdot \vec{A} \quad N.m^2/C$$

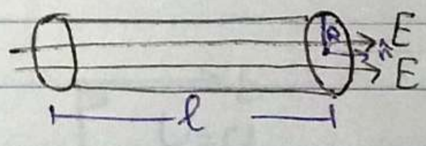
$$\Phi_E = EA\cos\theta \quad N.m^2/C$$

$$\Phi_E = \int \vec{E} \cdot d\vec{A}$$



• Sample problem 23.01 :

. Find Φ_E from each surface ?



→ $\Phi_{\text{right face}} = E\pi R^2 \cos\theta$, θ = zero

$\Phi_r = E\pi R^2$

→ $\Phi_{\text{left face}} = E\pi R^2 \cos 180$, $\theta = 180°$

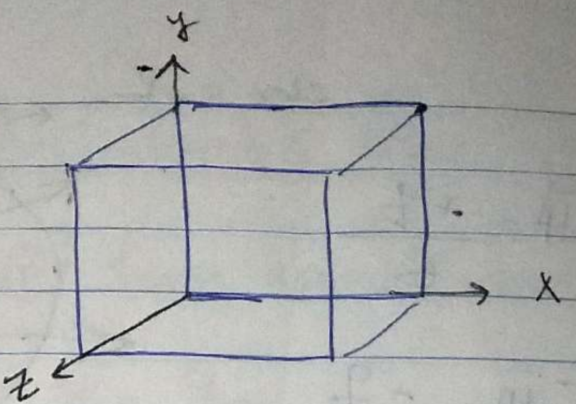
$\Phi_L = -E\pi R^2$

→ $\Phi_{\text{curved face}} = E\, 2\pi R l \cos\theta$, $\theta = 90°$

Φ_c = zero

• $\Phi_{\text{sylinder}} = \Phi_r + \Phi_L + \Phi_c$ = zero.

• example :



• Cube $l = 0.5m$

• A face Area $= 0.25\ m^2$

• $\vec{E} = 3\hat{i} + 4\hat{j} - 5\hat{k}$ N/C

⇒ Find Φ_E from each face?

• $\Phi_{front} = \vec{E}.\vec{A} = (3\hat{i}+4\hat{j}-5\hat{k}).(0.25\hat{k}) = -\frac{5}{4}\ N.m^2/C$

• $\Phi_{back} = \vec{E}.\vec{A} = (3\hat{i}+4\hat{j}-5\hat{k}).(-0.25\hat{k}) = \frac{5}{4}\ N.m^2/C$

• $\Phi_{right} = \vec{E}.\vec{A} = (3\hat{i}+4\hat{j}-5\hat{k}).(0.25\hat{i}) = \frac{3}{4}\ N.m^2/C$

• $\Phi_{Left} = \vec{E}.\vec{A} = (3\hat{i}+4\hat{j}-5\hat{k}).(-0.25\hat{i}) = -\frac{3}{4}\ N.m^2/C$

• $\Phi_{top} = \vec{E}.\vec{A} = (3\hat{i}+4\hat{j}-5\hat{k}).(0.25\hat{j}) = 1\ N.m^2/C$

• $\Phi_{bottom} = \vec{E}.\vec{A} = (3\hat{i}+4\hat{j}-5\hat{k}).(-0.25\hat{j}) = -1\ N.m^2/C$

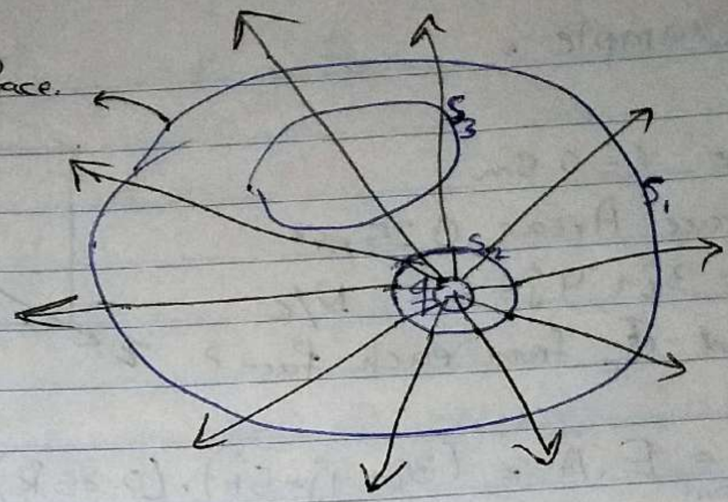
$$\Phi_{cub} = \Phi_f + \Phi_b + \Phi_r + \Phi_L + \Phi_t + \Phi_{bo} = \text{zero}.$$

• Gauss law :- Electric flux through a closed surface is equal to charge inside the Closed surface$/\epsilon_0$.

• $$\Phi_{E(\text{closed surface})} = \frac{q_{enc}}{\epsilon_0}$$

Thats mean a close surface.

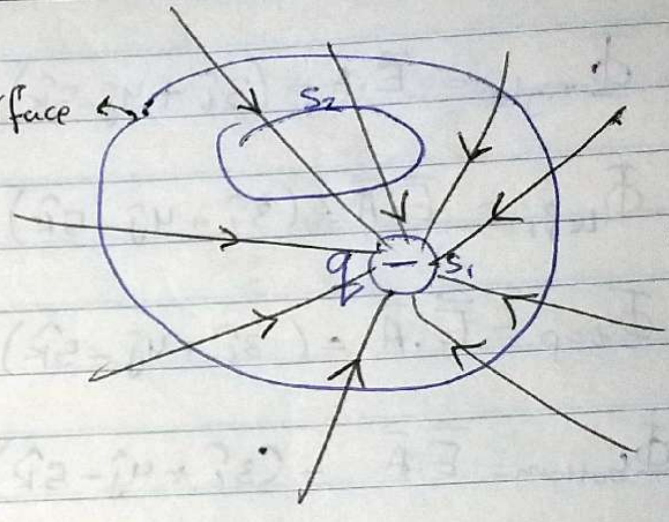
• $$\oint \vec{E}.\vec{dA} = \frac{q_{enc}}{\epsilon_0}$$



$\oint_{S_1} \vec{E} \cdot \vec{dA} = \frac{+q}{\epsilon_0}$

$\oint_{S_2} \vec{E} \cdot \vec{dA} = \frac{+q}{\epsilon_0}$

$\oint_{S_3} \vec{E} \cdot \vec{dA} = \text{zero}$



$\oint_{S_1} \vec{E} \cdot \vec{dA} = \frac{-q}{\epsilon_0}$

$\oint_{S_2} \vec{E} \cdot \vec{dA} = \cancel{\frac{q}{\epsilon_0}}\ \text{zero}$

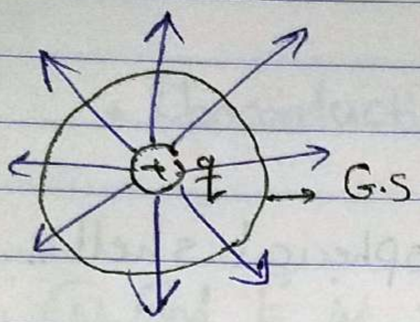
. لا يتغير التدفق داخل سطح مغلقه بخروج شحنة خارج السطح المغلق

. Gauss' law is useful in Calculating $\vec{E}$ of a continuous charge distribution in a system having a high sumetry

- spherical summetry
- Cylindrical Summetry
- Planer Summetry

• Gauss' law & Coulomb's law

• Find $\vec{E}$ at a distance (r) frome apoint charg= +q?



• $\oint \vec{E} \cdot d\vec{A} = \frac{q_{enc}}{\epsilon_0}$

$\vec{E} \int d\vec{A} = \frac{q_{enc}}{\epsilon_0}$

$\vec{E} \, 4\pi r^2 = \frac{q}{\epsilon_0}$

$\vec{E} = \frac{q}{\pi 4 \epsilon r^2} = \frac{kq}{r^2}$ —※

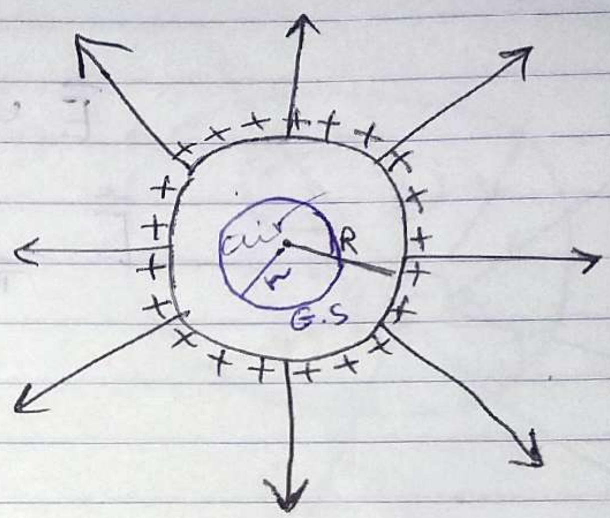
• Application on Gauss' law :-

$$\oint \vec{E} \cdot d\vec{A} = \frac{q_{enc}}{\epsilon_0}$$

1 Spherial symmetry :-

a) $\vec{E}$ due to a Uniformly Charged spherical shell :-

① Find $\vec{E}$ at $r < R$?

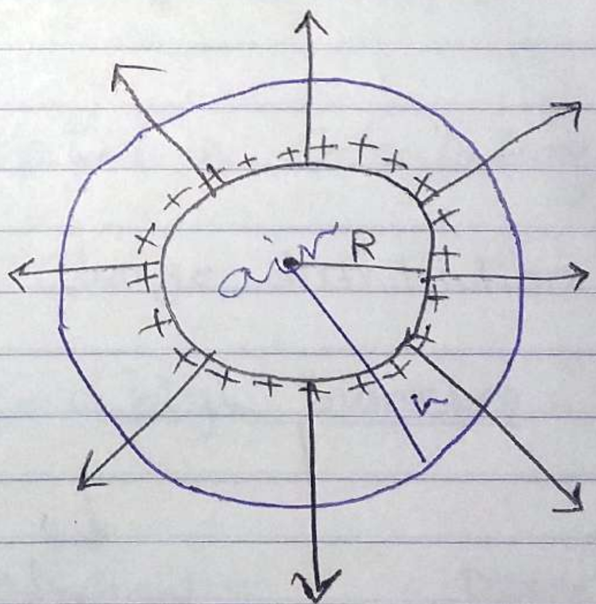


$$\oint \vec{E} \cdot d\vec{A} = \frac{q_{enc}}{\epsilon_0}$$

$$E \, 4\pi r^2 = 0$$

$$, E = 0 \quad \text{at } r < R$$

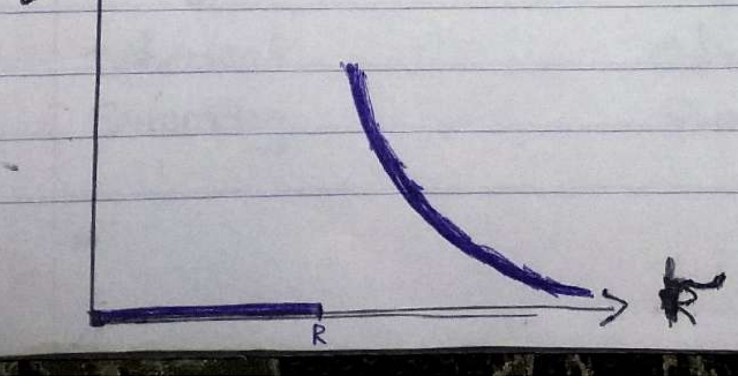
② Find $\vec{E}$ at $r \geq R$?



$$\oint \vec{E} \cdot d\vec{A} = \frac{q_{enc}}{\epsilon_0}$$

$$E \, 4\pi r^2 \cos 0 = \frac{q_{enc}}{\epsilon_0}$$

$$\Rightarrow E = \frac{q}{\epsilon_0 4\pi r^2} \quad , \text{at } r \geq R$$



b) $\vec{E}$ due to a Uniformaly Charged NonConducting sphere :-

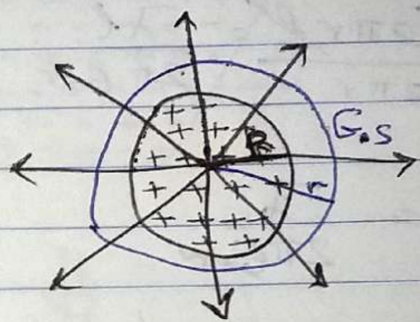
• Nonconducting sphere → radius $= R$; Charge $= q$; $\rho = \dfrac{q}{\frac{4}{3}\pi R^3}$

① Find $\vec{E}$ at $r \geq R$?

$$\oint \vec{E} \cdot d\vec{A} = \frac{q_{enc}}{\epsilon_0}$$

$$\vec{E}\, 4\pi r^2 = \frac{q}{\epsilon_0}$$

$$\vec{E} = \frac{q}{4\pi r^2 \epsilon_0} \quad , \quad r \geq R$$

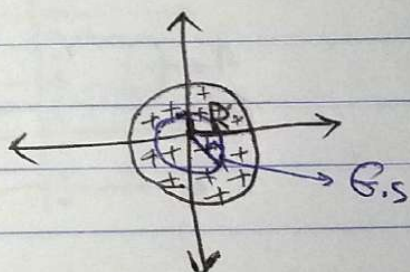


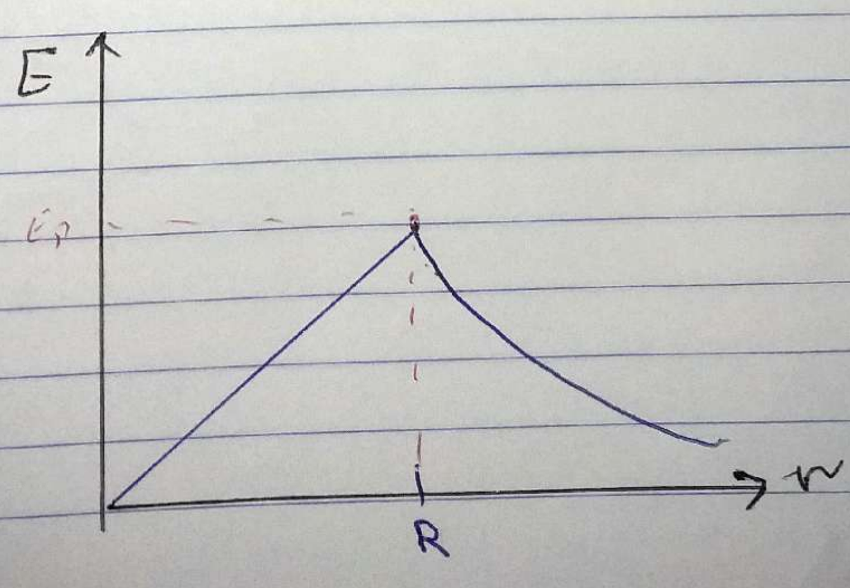
② Find $\vec{E}$ at $r < R$?

$$\oint \vec{E} \cdot d\vec{A} = \frac{q_{enc}}{\epsilon_0}$$

$$\vec{E}\, 4\pi r^2 = \frac{1}{\epsilon_0} \rho \left(\frac{4}{3}\pi r^3\right)$$

$$\vec{E} = \frac{\rho r}{3\epsilon_0} \quad , \quad r \leq R$$





2 Sylindrical symmetriy :-

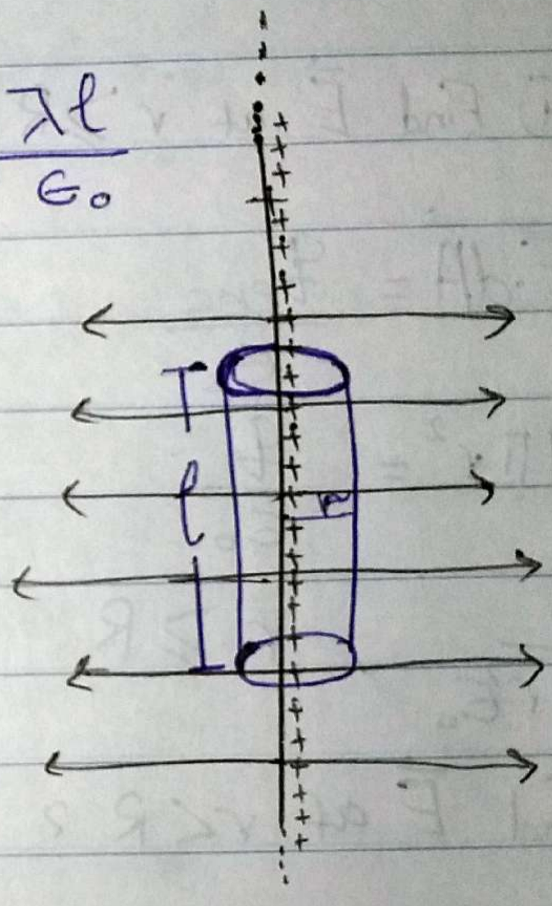
$\vec{E}$ due to a Uniformly Charged very (long wire) or (infinite wire):-

- Find $\vec{E}$ near a Charged infinit rod (λ) ?

$$\oint \vec{E} \cdot d\vec{A} = \frac{q_{enc}}{\epsilon_0} \Rightarrow \oint \vec{E} \cdot d\vec{A} = \frac{\lambda \ell}{\epsilon_0}$$

$$\frac{E\, 2\pi r \ell}{2\pi r} = \frac{\lambda \ell}{2\pi \epsilon_0 r}$$

$$E = \frac{\lambda}{2\pi \epsilon_0 r}$$



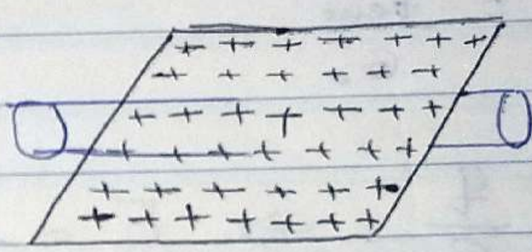
[3] planer summetry :-

$\vec{E}$ due to a uniformly Charged thin infint sheet :-

(Non Conducting infinite Sheet <very thin>) :-

• surface Charg density = σ on one face.

• G.S is a sylinder Crossing the sheet from the two Side :-



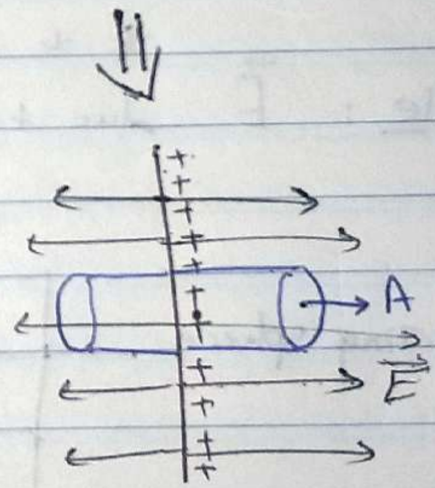
$$\oint E.dA = \frac{q_{enc}}{\epsilon_0}$$

$$\Rightarrow EA\cos\theta + EA\cos\theta = \frac{\sigma A}{\epsilon_0}$$

$$2EA = \frac{\sigma A}{\epsilon_0}$$

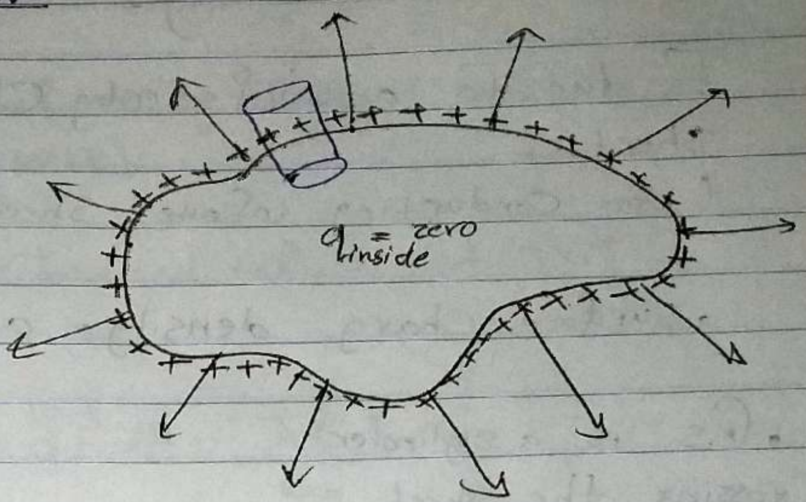
$$E = \frac{\sigma}{2\epsilon_0}$$

infinite Sheet



• Charge Conducter :

• q rests on the outer
→ surface = 0
→ $E = 0$



• Find E near the Outer surface

$$\oint \vec{E}\cdot d\vec{A} = \frac{q_{enc}}{\epsilon_0}$$

$$EA = \frac{q}{\epsilon_0} \Rightarrow E = \frac{\sigma}{\epsilon_0}$$

↳ near the outer surface of a conducter.

* * * * *

• example : $\vec{E}$ due to a charge conducting sphere?

• conducting sphere
- → radius
- → Charge
- → $\rho = 0$ → الشحنة مستقرة ، [illegible]
- → $\sigma = \frac{q}{4\pi R^2}$

• $E = 0$, at $r < R$

• $E = \frac{q}{4\pi\epsilon_0 r^2}$, at $r \geq R$